

INDIAN STATISTICAL INSTITUTE

Midsem Examination: 2019-20 (First Semester)
Bachelor of Mathematics (B. Math.) III Year
Probability Theory III

Teacher: Parthanil Roy

Date: 13/09/2019

Maximum Marks: 30

Duration: 10:00 am - 12:30 pm

Note:

- Please write your roll number on top of your answer paper.
- You may use any theorem proved or stated in the class but do not forget to quote the appropriate result.
- You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc. If you are caught using any, you will get a zero in this examination.
- Failing to follow the examination guidelines, copying in the examination, rowdyism or some other breach of discipline or unlawful/unethical behavior, etc. are regarded as unsatisfactory conduct. Any student caught cheating or violating examination rules will get a zero in this examination.

1. Suppose $\{N_t\}_{t \geq 0}$ is a homogeneous Poisson process with rate α as described in the class. Fix $k, n \in \mathbb{N}$ and $0 < s_1 < s_2 < \dots < s_k < u$. Find the conditional distribution of $(N_{s_1}, N_{s_2}, \dots, N_{s_k})$ given $N_u = n$. What is your answer if $n = 0$? [9 + 1 = 10]
2. Let $\{X_n\}_{n \geq 0}$ be the simple random walk on the free group F_2 starting from e (as described in the class). Suppose $|X_n|$ denotes the Cayley graph (with respect to standard generators) distance from X_n to e . Show that $\{|X_n|\}_{n \geq 0}$ is a Markov chain on $\mathbb{N} \cup \{0\}$. Find its initial distribution and transition probability matrix. [5 + 2 + 3 = 10]
3. Suppose two types of passengers arrive in an auto rickshaw stand with unlimited supply of auto rickshaws. The first type of passengers will wait patiently till *four passengers* (= capacity of an auto rickshaw) arrive and share the auto rickshaw. The second type will simply reserve the auto rickshaw and go away immediately. Assume that these two types of passengers arrive independently according to Poisson processes with rates 10 and 5 per hour, respectively. Priorities are given to second type of passengers even if a few passengers of the first type are waiting in the queue. However, while each of the first type of passengers pay Rs. 10 for the trip, the second type of passengers are charged Rs. 60. Assume also that no time is lost in passengers getting into the auto rickshaw, the driver takes the money from the passenger(s) and departs immediately without wasting any time. Given that exactly 6 passengers arrive during 9 : 00 am - 9 : 30 am, compute the expected total earning of auto rickshaw drivers in that time-span. [10]

Wish you all the best